

Name of Course	: CBCS-2 (LOCF) B.Sc. (H) Mathematics
Unique Paper Code	: 32351102
Name of Paper	: C2-Algebra BMATH102
Semester	: I
Duration	: 3 hours
Maximum Marks	: 75 Marks

Attempt any four questions. All questions carry equal marks.

- Solve the equations:
  - $6x^4 - 13x^3 - 35x^2 - x + 3 = 0$  given that one root is  $2 - \sqrt{3}$ .
  - $2x^3 - 3x^2 - 11x + 6 = 0$  given that all roots are rational.
  - $x^4 - 8x^3 + 14x^2 + 8x - 5 = 0$  given that roots are in arithmetic progression.
- Find  $|z|$ ,  $\arg z$ ,  $\text{Arg } z$ ,  $\arg(-z)$  and  $\arg \bar{z}$  for
 
$$z = (1 + i)^6 (2\sqrt{3} + 2i)^8.$$
 Solve the equation  $z^4 = -7 + 24i$ .
- Prove that the following relations ' $\sim$ ' defined on  $\mathbb{R}^2$  are equivalence relations. Also give the interpretation of the equivalence classes in each case.
  - $(a, b) \sim (c, d)$  iff  $3a^2 + 4b^2 = 3c^2 + 4d^2$ .
  - $(a, b) \sim (c, d)$  iff  $2a + 5b = 2c + 5d$ .
 Let  $A = \{a, b, c\}$ . List all the equivalence relations on  $A$ .
- For what values of  $x$  the graph of the following functions break (jump):
  - $y = \lfloor 4x - 3 \rfloor$  where  $\lfloor \cdot \rfloor$  denotes floor function.
  - $y = \lceil \frac{x}{3} + 2 \rceil$  where  $\lceil \cdot \rceil$  denotes ceiling function.
 Find the greatest common divisor of 94 and 120. Express it in the form  $94m + 120n$ , for some integers  $m$  and  $n$ .  
 Using principle of mathematical induction, prove that  $7^n - 1$  is divisible by 6 for  $n \geq 1$ .
- Let  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be defined by
 
$$T(x, y, z) = (-x + 4y + 5z, x + z, 2y + z).$$
 Check whether  $T$  is a linear transformation or not. Find  $[T]_\beta$ , where  $\beta$  is the standard ordered bases for  $\mathbb{R}^3$ . Find the eigenvalues and the eigenvectors associated to  $[T]_\beta$ .
- Solve the following system of linear equations:
 
$$\begin{aligned} x + 3y + 2z &= 3 \\ 2x + y + 4z &= 2 \\ 3x + 2y + 7z &= 5 \end{aligned}$$
 by reducing it to matrix form  $AX = B$ . Find the basis and dimension for the null space of  $A$ . Find row space of  $A$  and also find the rank of the matrix  $A$ .